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Involution and Body Force Method for Solution of Half Plane Problems with Sub-Surface Hole

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Abstract:
Flamant problem plays significance in practical problems such as bridges. It is concentrated load acting on the free boundary of semi-infinite plate. Solution for flamant problem with hole has been obtained by classical method such as involution technique . Body force method, a semi-numerical method based on principle of superposition is used to obtain solutions [1,2]. In the the previous works the actual condition is treated as an imaginary condition i.e. the semi-infinite plate with hole is treated as a plate without hole; the actual hole is regarded as imaginary on whose periphery boundary forces are applied. The problem is solved by superimposing the stress fields of the boundary forces and concentrated force acting at an arbitrary point to satisfy the prescribed boundary conditions so that the stress condition of the actual plate is approximately equal to that of the imaginary plate. In the present work, complex potentials of involution technique for the Kelvin-type problem (In plane concentrated load acting in the infinite plate with hole) [3,4] is used as fundamental solution.The flamant problem with hole is shown in Fig.1.

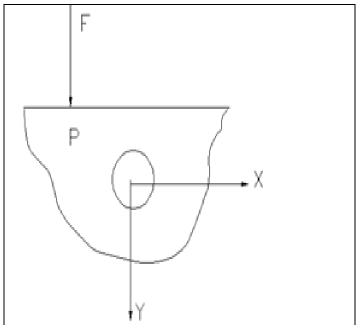

Fig.1. Flamant problem with hole

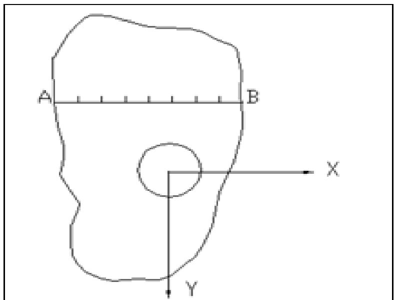

Fig.2 Divisions on infinite plate

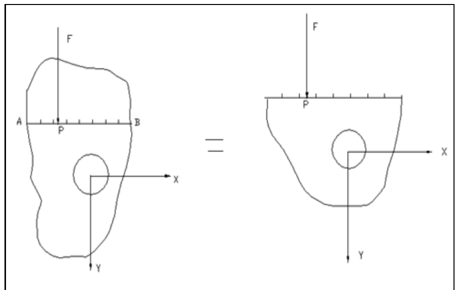

Fig.3 Equivalance of flamant problem with infinite plate (Body forces on infinite plate is not shown)

As contrast to dividing the imaginary hole into number of divisions as in the previous works, the imaginary free boundary (AB} of infinite plate with hole acted upon by a concentrated load 'F' at "P" is divided into number of equal divisions as shown in Fig.2. On each division unit concentrated load in x and y directions are applied and resultant forces along each division in x and y directions are calculated due to these unit concentrated loads. The boundary conditions require that these resultant forces along each segment are nullified due to resultant forces along each segments created by concentrated load acting on the boundary. By this ,the imaginary free boundary of infinte plate becomes stress free and becomes free edge of semi-infinite plate as shown in Fig,2.The body forces are calculated which are to be applied at the midpoint of each divisions. Now the free boundary becomes stress free and it is equivalent to flamant problem with hole. Hoop stresses, radial stresses and tangential stresses are calculated around the hole and compared with involution technique. It is found that the results are more closure to involution technique than the previous work wherein imaginary hole is descretised. In the present method, singularities near the hole boundary is nullified.

1. Introduction

Stress analysis of components is important in the design and development of machines and structures. For determining the stress distribution in a body, fourth order partial differential equation (Bi-harmonic equation) for the stress function is to be solved. Closed form solutions are available for a simple geometry, load and boundary conditions. Presence of geometrical discontinuity is a big challenge. A slight change in the shape and geometry discontinuity will affect the problem entirely different. Timoshenko and Goodier [5], Love [6] and Muskhelishvili [7] have developed methods to solve a number of problems of practical relevance. Complex variable approach is well suited for stress analysis of the problem and discontinuity with holes, cracks etc. Honein and Hermann [8] employed complex variable approach to solve the problem of infinite plate with circular hole / inclusions subjected to in plane concentrated load. Ramakrishna [3] extended the approach to solve the problem of half plane. This problem becomes very difficult when the geometry of discontinuity becomes complex. The boundary (body) force method proposed by H.Nisitani which is hybrid theoretical and numerical method is found to be versatile method when discontinuities like holes or cracks are present.

2. Body Force Method

This method is based on the fundamental solution of concentrated load acting in an infinite plate with holes. To start with infinite plate is considered with hole having Kelvin-type of loading (inplane concentrated load). The actual free boundary is considered to be an imaginary free boundary as shown in Fig.1. The imaginary free boundary is divided into number of equal divisions. At the mid-point of each division, unit concentrated load in x and y directions are applied. The resultant forces due to all concentrated loads at all divisions are calculated from the complex potentials of fundamental solution. These resultant forces acting on each divisions of imaginary free boundary are nullified due to concentrated load acting at point P as shown in Fig . Finally a square matrix with influence coefficients (resultant force at N^{th} division due to concentrated load acting at M^{th} division), column matrix with unknown body forces and column matrix containing resultant forces on all divisions due concentrated load is obtained. After solving this, body forces in x-direction and y-directions are obtained. If these body forces are applied at mid-point of each division, the imaginary free boundary becomes stress free and it will be equivalent to flamant problem with hole as shown in Fig.3. The stress at any point is given by the sum of the stresses due to all body forces and concentrated load acting at point 'P' (principle of super position).

3. Calculation of Unknown Body Forces

Influence coefficients P_{XM}^{XN}, P_{YM}^{XN} *etc.*. which are the resultant forces in 'x' and 'y' directions respectively at M^{th} division due to unit force acting at the mid-point of N^{th} division in 'x' direction are calculated. Conditions that make the mid-point of the M^{th} interval free from stresses by using the body forces ρ_{XN} *and* ρ_{YN} (which are unknowns to start with) and the influence coefficients are developed. The following sets of 2M linear equations are formed which expressed in matrix form as follows.

$$\begin{bmatrix} P_{X1}^{X1} & P_{X1}^{X2}\ldots\ldots P_{X1}^{XN} & P_{X1}^{Y1} & P_{X1}^{Y2}\ldots\ldots P_{X1}^{YN} \\ P_{Y1}^{X1} & P_{Y1}^{X2}\ldots\ldots P_{Y1}^{XN} & P_{Y1}^{Y1} & P_{Y1}^{Y2}\ldots\ldots P_{Y1}^{YN} \\ P_{X2}^{X1} & P_{X2}^{X2}\ldots\ldots & & \\ P_{Y2}^{X1} & P_{Y2}^{X2}\ldots\ldots & & \\ \ldots\ldots & & & \\ \ldots\ldots & & & \\ P_{XM}^{X1}\ldots\ldots & & & P_{XM}^{YN} \\ P_{YM}^{X1}\ldots\ldots & & & P_{YM}^{YN} \end{bmatrix} \begin{bmatrix} \rho_{X1} \\ \rho_{X2} \\ \rho_{X3} \\ \rho_{X4} \\ . \\ \rho_{XN} \\ \rho_{Y1} \\ \rho_{YN} \end{bmatrix} = \begin{bmatrix} -P_{X1}^{A} \\ -P_{Y1}^{A} \\ -P_{X2}^{A} \\ -P_{Y2}^{A} \\ \\ . \\ -P_{XM}^{A} \\ -P_{YN}^{A} \end{bmatrix}$$

The matrix is solved for unknown body forces. The stresses at an arbitrary point can be expressed in terms of linear combination of ρ_{XN} *and* ρ_{YN}. The complex potentials for infinite plate with hole having concentrated in planer load is used. when concentrated force acts at an arbitrary point $z_0 = x_0 + iy_0$ we have [3],

$$S = \left(\frac{a^2}{z} - \bar{z}_0 \right) \tag{1}$$

$$\phi(z) = -\frac{F}{2\pi(1+\kappa)}\left[e^{i\gamma}\log(z-z_0) + \frac{\kappa e^{i\gamma}}{\alpha}\log S - \frac{(z-z_0)e^{-i\gamma}}{\alpha S} - \frac{ze^{-i\gamma}}{\alpha \bar{z}_0}\right] \quad (2)$$

$$\kappa e^{-i\gamma}\log(z-z_0) = A \quad (3)$$

$$\frac{\bar{z}_0 e^{i\gamma}}{(z-z_0)} = B \quad (4)$$

$$\alpha e^{-i\gamma}\log S = C \quad (5)$$

$$\frac{\kappa e^{i\gamma}}{S} + \frac{(z-z_0)e^{-i\gamma}}{S^2} + \frac{z^2 e^{-i\gamma}}{a^2} = D \quad (6)$$

$$\psi(z) = \frac{F}{2\pi(1+\kappa)}\left[A + B + C - \frac{a^4}{\alpha z^3}\left\{D\left(\frac{1}{S} + \frac{1}{\bar{z}_0}\right)\right\} - \frac{a^2 e^{i\gamma}}{zz_0}\right] \quad (7)$$

Differentiating equation (2) twice, we get $\phi'(z) and \phi''(z)$.

Differentiating equation (7) once, we get $\psi'(z)$

The components of the stress and the resultant force can be expressed in terms of the complex potentials as

$$\sigma_x + \sigma_y = 2\left[\phi'(z) + \bar{\phi}'(z)\right] \quad (8)$$

$$\sigma_x - \sigma_y + 2i\tau_{xy} = -2\left[z\bar{\phi}''(z) + \bar{\psi}'(z)\right] \quad (9)$$

$$p_x + ip_y = -i\left[\phi(z) + z\bar{\phi}'(z) + \bar{\psi}(z)\right]_A^B \quad (10)$$

When equation (2) and (7) and their corresponding derivatives are substituted in equation (8) and (9), Cartesian stresses and polar stresses can be calculated. when equation (2) and (7) and their corresponding derivatives are substituted in equation (10), p_x, p_y can be obtained. The symbols p_x, p_y represent the components of the resultant force along the path from one end of division to another end of division. When the concentrated force acts at point ‘ P ‘on the imaginary free boundary, the same equations are used to calculate resultant forces p_x, p_y along each division, which nullifies the resultant forces created by unit concentrated loads. By this, imaginary free boundary becomes stress free and becomes Flamant problem with hole.

4. Involution and Kelvin Transformation

The general method of solution of an elastic inclusion problem employing involution and Kelvin transformation approach by Honein and Herrmann [8] and Ramakrishna [3] is made use for flamant problem. This method is used for comparison with body force method. The potentials for the problem of flamant with circular hole and subjected to the point load are obtained. Software is developed to calculate the same. The results are compared with BFM software developed.

5. Numerical Results

The circular hole of 2mm radius is considered in flamant problem subjected to concentrated load 'F' of 100 N at distance Xo=-10 and Yo=-10 from the origin as shown in Fig.1 The radial, hoop and tangential stresses along radial direction at angles 5degree , 30degree and 178degreeare computed and compared with involution technique. Stresses along 5-degree radial line are shown in Fig4, Fig5 and fig6 , stresses along 8-degree radial line are shown in Fig7 Fig8 and Fig9.and strsses along 178degree are shown in Fig10 ,Fig11 .and Fig12 .Initially, mesh size is taken as 20 divisions.convergent study is carried out by refining the mesh to 40,60 and 80 divisions. The convergent results of polar stresses at a radius of 2.5 units and 8degree clockwise from positive X-axis is presented.

DIVISIONS	BFM(HOLE DIVISIONS)	BFM(EDGE DIVISIONS)	INVOLUTION TECHNIQUE
20	-0.296	-0.281	-0.267
40	-0.294	-0.272	
60	-0.290	-0.269	
80	-0.289	-0.268	

Table 1: Values Of Radial Stresses

DIVISIONS	BFM(HOLE DIVISIONS)	BFM(EDGE DIVISIONS)	INVOLUTION TECHNIQUE
20	-0.643	-0.623	-0.619
40	-0.639	-0.621	
60	-0.638	-0.622	
80	-0.636	-0.621	

Table 2: Values Of Hoop Stresses

DIVISIONS	BFM(HOLE DIVISIONS)	BFM(EDGE DIVISIONS)	INVOLUTION TECHNIQUE
20	-1.172	-1.181	-1.189
40	-1.171	-1.183	
60	-1.178	-1.184	
80	-1.176	-.1.191	

Table 3: Values Of Shear Stresses

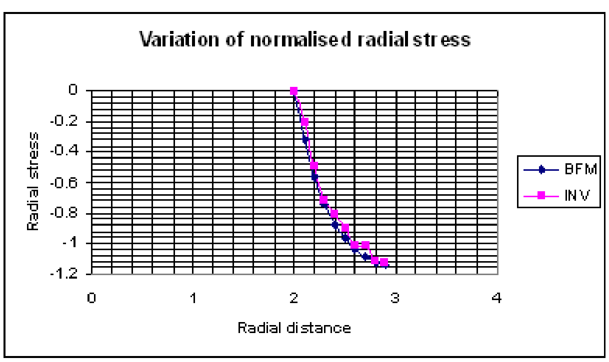

Figure 4

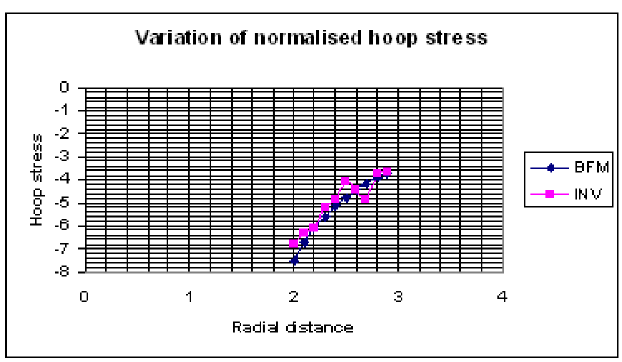

Figure 5

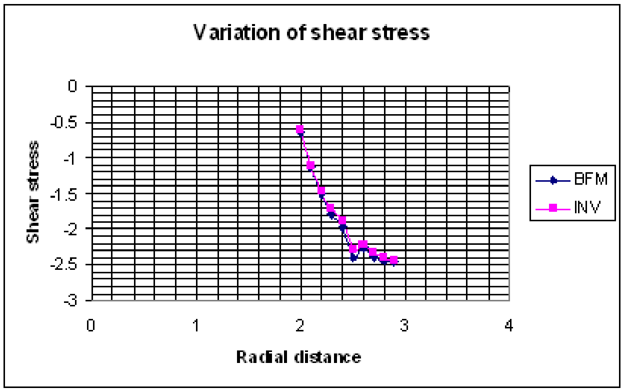

Figure 6

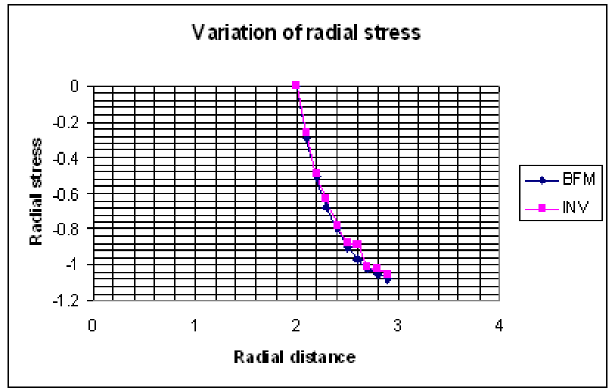

Figure 7

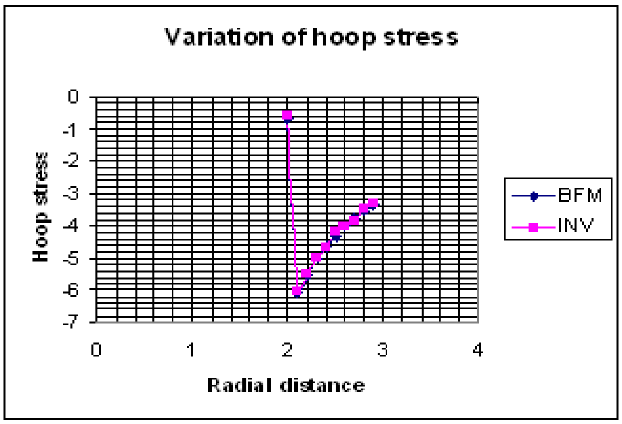

Figure 8

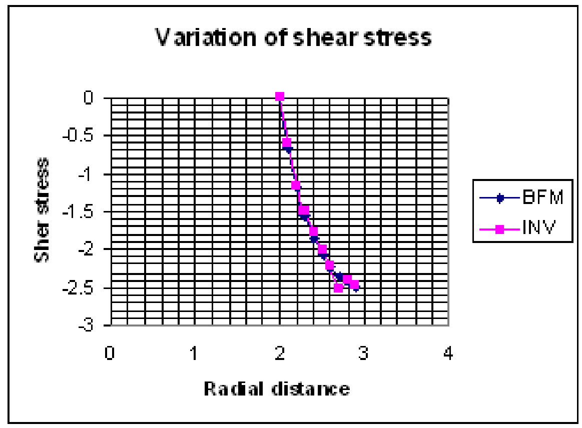

Figure 9

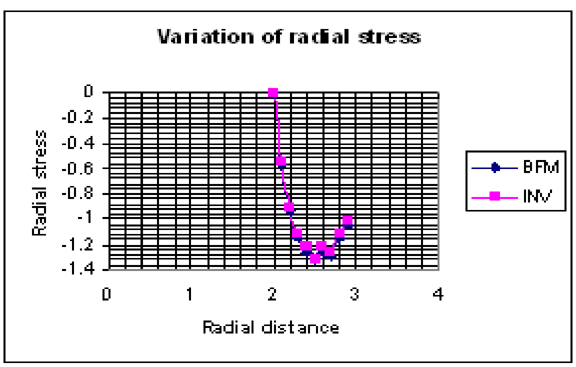

Figure 10

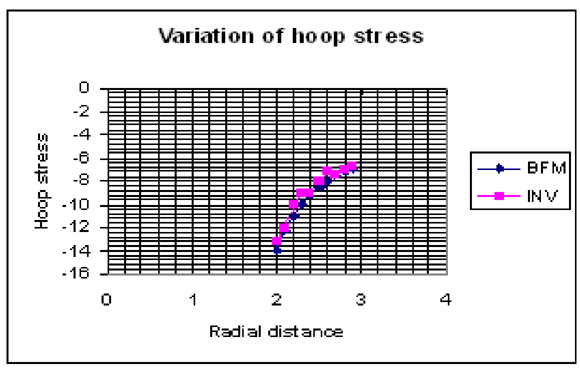

Figure 11

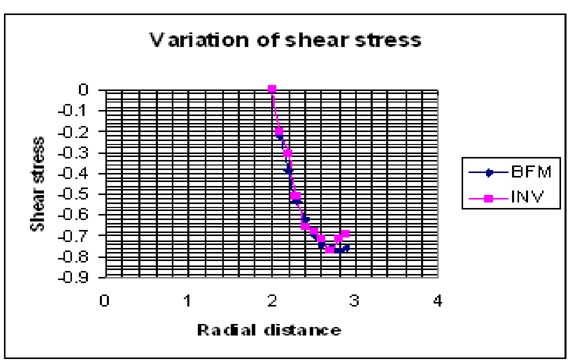

Figure 12

6. Conclusion

The results obtained for flamant problem from body force method by using fundamental solution for concentrated load acting in infinite plate with hole (complex potential from involution technique) is very close with the results of involution technique than the the results from body force method by meshing hole boundary[9,10,11,12,13].It is also observed that by meshing the hole boundary ,singularity exists which can be avoided by the present method used in this work..Also,melan problem (concentrated load acting inside the free edge of semi-infinte plate with hole) [10] can be solved without making use of melan potentials.

7. References

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8. Nomenclature

BFM: Body Force Method

γ: Angle of inclination of load w.r.t x-axis

κ: (3--4ν) for plane strain
(3-ν)/(1+ν) for plane stress

Z: $X + iY$

$\phi(Z), \psi(Z)$: Complex potentials, functions of Complex variable $X + iY$

ρ_{X1}: Body force in X -direction

ρ_{Y1}: Body force in Y -direction

z_0 : $x_0 + iy_0$

$\bar{z}$: Conjugate of $z = (x - iy)$

$\bar{z}_{01}$: Conjugate of $z_{01} = x_{01} - iy_{01}$