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Unsteady MHD Free Convective Viscous Incompressible Fluid Flow Past an Accelerated Vertical Plate with Ohmic Heating

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Abstract:
In this paper, finite difference solution of an unsteady MHD free convective fluid flow of a viscous fluid past an accelerated vertical plate in the presence of rotation is thoroughly investigated. The plate temperature as well as concentration level near the plate are raised linearly with time.The effects of the various parameters on the velocity, temperature and concentration profiles are presented graphically and values of skin– friction coefficient, Nusselt number and Sherwood number for various values of physical parameters are also precisely discussed.

Key words: *Primary Velocity, Angular Velocity, MHD free convective flow, Finite difference method*

1. Introduction

Magnetohydrodynamics plays an important role in Petroleum industries, Geophysics, and Meteorology, etc. The hydromagnetic convection with heat and mass transfer in porous medium has been studied as it has essentially become important in the design of MHD generators and accelerators in geophysics, in design of underground water energy storage system, soil-sciences, astrophysics, nuclear power reactors and so on. Magnetohydrodynamics is currently undergoing a period of great enlargement and differentiation of subject matter. Gupta and Pop (1) studied free convection on flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat. Kafousias N. G. and Raptis A. (2) extended the above problem to include mass transfer effect subjected to variable suction and injection. Raptis et al. (3) studied the free convective flow past an accelerated vertical plate subjected to variable suction and uniform heat flux in the presence of magnetic field.

Soundalgekar (4) investigated the mass transfer effect on the flow past a uniformly accelerated vertical plate. Singh and Singh (5) studied the mass transfer effect on the flow past an accelerated vertical plate subjected to variable suction and uniform heat flux. Basantha and Ravindra Prasad (6) analyzed the effect of mass transfer on the flow past an accelerated infinite vertical plate with heat sources. Singh (7) studied MHD flow past an impulsively started vertical plate in a rotating fluid. The effect of rotation on hydromagnetic free convective flow past an accelerated isothermal vertical plate was studied by Raptis et al. (8). Muthucumaraswamy et al. (9) studied rotation effects on MHD flow past an accelerated isothermal vertical plate with uniform mass diffusion. MHD flow past an accelerated vertical plate with variable heat and mass diffusion in the presence of rotation is analyzed by Muthucumaraswamy et al. (10).

In this paper, an MHD viscous flow past an accelerated vertical plate through porous medium with variable heat and mass diffusion in the presence of rotation is thoroughly studied. Such a study is essentially useful in magnetic control of molten iron flow in the Steel Industry and liquid metal cooling in nuclear reactors, magnetic suppression of molten semi–conducting materials and meteorology. The governing equations are solved by using finite difference technique.The effects of the various parameters on the velocity, temperature and concentration profiles are presented graphically and values of skin– friction coefficient, Nusselt number and Sherwood number for various values of physical parameters are also discussed through tables.

2. Mathematical Analysis
An unsteady hydromagnetic flow of a viscous incompressible fluid past a uniformly accelerated motion of an infinite vertical plate when the fluid and the plate rotate as a rigid body with a uniform angular velocity Ω about z' axis in the presence of an imposed uniform magnetic field normal to the plate is considered under the following assumptions:

- Initially the temperature of the plate and concentration near the plate are assumed to be T'_∞ and C'_∞.
- At time $t' > 0$, the plate starts moving with a velocity $u = u_0 t'$ in its own plane
- The temperature of the plate as well as wall concentration near the plate is raised linearly with time.
- Since the plate occupying the plane $z' = 0$ is of infinite extent, all the physical quantities depend only on z' and t'
- Magnetic dissipation (Joules heating of the fluid) is considered.
- The induced magnetic field is neglected.

Under the assumptions the governing equations, the initial and boundary conditions of the flow become

$$\frac{\partial u'}{\partial t'} - 2\,\Omega' v' = \nu \frac{\partial^2 u'}{\partial z'^2} - \left(\frac{\sigma B_0^2}{\rho} + \frac{\nu}{K'}\right) u' + g\,\beta\left(T' - T'_\infty\right) + g\,\beta^*\left(C' - C'_\infty\right) \qquad (1)$$

$$\frac{\partial v'}{\partial t'} + 2\,\Omega' u' = \frac{\partial^2 v'}{\partial z'^2} - \left(\frac{\sigma B_0^2}{\rho} + \frac{\nu}{K'}\right) v' \qquad (2)$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_P}\frac{\partial^2 T'}{\partial z'^2} + \frac{\mu}{\rho C_P}\left(\frac{\partial u'}{\partial z'}\right)^2 - \frac{\sigma B_0^2}{\rho C_P}\left(u'\right)^2 \qquad (3)$$

$$\rho C_P \frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial z'^2} \qquad (4)$$

The initial and boundary conditions are

$$t' \le 0 : \quad u' = 0, \quad v' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \quad \textit{for all } Z'$$

$$t' > 0 \quad \begin{cases} u' = u_0 t', \quad v' = 0, \quad T' = T'_\infty + \left(T'_w - T'_\infty\right) A t' & \\ C' = C'_\infty + \left(C'_w - C'_\infty\right) A t' & \textit{at } Z' = 0 \\ u \to 0, \quad T' \to T'_\infty, \quad C' \to C'_\infty & \textit{as } Z' \to \infty \end{cases} \qquad (5)$$

Introducing the non – dimensional quantities

$$U = \frac{u'}{\left(\nu u_0\right)^{1/3}} \qquad V = \frac{v'}{\left(\nu u_0\right)^{1/3}} \qquad t = t'\left(\frac{u_0^2}{\nu}\right)^{1/3} \qquad z = z'\left(\frac{u_0}{\nu^2}\right)^{1/3}$$

$$\theta = \frac{T' - T'_\infty}{T'_w - T'_\infty} \qquad \phi = \frac{C' - C'_\infty}{C'_w - C'_\infty} \qquad A = \left(\frac{u_0}{\nu^2}\right)^{1/3} \qquad (6)$$

Due to the physical parameters, defined in the nomenclature, the equations (1) to (4) and the boundary conditions (5), the governing equations and the initial and boundary conditions become in the non – dimensional form as

$$\frac{\partial u}{\partial t} - 2\Omega v = \frac{\partial^2 u}{\partial z^2} - \left(M + \frac{1}{K}\right) u + Gr\,\theta + Gc\,\phi \tag{7}$$

$$\frac{\partial v}{\partial t} + 2\Omega u = \frac{\partial^2 v}{\partial z^2} - \left(M + \frac{1}{K}\right) v \tag{8}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial z^2} + Ec\left(\frac{\partial u}{\partial z}\right)^2 - M\,Ec\,u^2 \tag{9}$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{\mathrm{Sc}}\frac{\partial^2 \phi}{\partial z^2} \tag{10}$$

$$\begin{aligned} & t \le 0 \qquad u = 0,\ v = 0,\ \theta = 0,\ \phi = 0 \qquad \text{for all } z \\ & t > 0 \quad \begin{cases} u = t, & v = 0, \quad \theta = t, \quad \phi = t \qquad \text{for } z = 0 \\ u \to 0, & v \to 0, \quad \theta \to 0, \quad \phi \to 0 \qquad \text{as } z \to \infty \end{cases} \end{aligned} \tag{11}$$

3. Finite Difference Technique

The governing equations (8) – (10) are coupled non – linear partial differential equations and are solved, by using the initial and boundary conditions (11). Exact or approximate solutions are not possible for this set of equations; hence an explicit finite difference method is used. The finite difference scheme of governing equations (8) – (10) and the conditions (11) are given by

$$\frac{\partial u}{\partial t} = \frac{u(i,\ j+1) - u(i,\ j)}{\Delta t} \qquad \frac{\partial \theta}{\partial t} = \frac{\theta(i,\ j+1) - \theta(i,\ j)}{\Delta t}$$

$$\frac{\partial \phi}{\partial t} = \frac{\phi(i,\ j+1) - \phi(i,\ j)}{\Delta t} \qquad \frac{\partial u}{\partial z} = \frac{u(i+1,\ j) - u(i,\ j)}{\Delta z}$$

$$\frac{\partial \theta}{\partial z} = \frac{\theta(i+1,\ j) - \theta(i,\ j)}{\Delta z} \qquad \frac{\partial \phi}{\partial z} = \frac{\phi(i+1,\ j) - \phi(i,\ j)}{\Delta z}$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{u(i+1,\ j) - 2u(i,\ j) + u(i-1,\ j)}{\Delta z^2}$$

$$\frac{\partial^2 \theta}{\partial z^2} = \frac{\theta(i+1,\ j) - 2\theta(i,\ j) + \theta(i-1,\ j)}{\Delta z^2}$$

$$\frac{\partial^2 \phi}{\partial z^2} = \frac{\phi(i+1,\ j) - 2\phi(i,\ j) + \phi(i-1,\ j)}{\Delta z^2}$$

Using the above finite difference scheme, the governing equations (8) – (10) become

$$\begin{aligned} \frac{u(i,\ j+1) - u(i,\ j)}{\Delta t} - 2\Omega v(i,\ j) = & \frac{u(i+1,\ j) - 2u(i,\ j) + u(i-1,\ j)}{\Delta z^2} \\ & - \left(M + \frac{1}{K}\right) u(i,\ j) \\ & + Gr\ \theta(i,\ j) + Gc\ \phi(i,\ j) \end{aligned} \tag{12}$$

$$\begin{aligned} \frac{v(i,\ j+1) - v(i,\ j)}{\Delta t} + 2\Omega\ u(i,\ j) = & \frac{v(i+1,\ j) - 2v(i,\ j) + v(i-1,\ j)}{\Delta z^2} \\ & - \left(M + \frac{1}{K}\right) v(i,\ j) \end{aligned} \tag{13}$$

$$\frac{\theta(i,\ j+1)-\theta(i,\ j)}{\Delta t} = \frac{1}{\Pr}\,\frac{\theta(i+1,\ j)-2\theta(i,\ j)+\theta(i-1,\ j)}{\Delta z^2} + Ec\left(\frac{u(i+1,\ j)-u(i,\ j)}{\Delta z}\right)^2 - M\,Ec\left[\ u(i,\ j)\ \right]^2 \tag{14}$$

$$\frac{\phi(i,\ j+1)-\phi(i,\ j)}{\Delta t} = \frac{1}{Sc}\,\frac{\phi(i+1,\ j)-2\phi(i,\ j)+\phi(i-1,\ j)}{\Delta z^2} \tag{15}$$

The initial conditions are represented as

$$u(i,\ 0)=0,\quad \theta(i,\ 0)=0,\quad \phi(i,\ 0)=0 \qquad \text{for all } i \tag{16}$$

The boundary conditions are

$$\begin{aligned} &u(0,\ j)=t,\quad v(0,\ j)=0,\quad \theta(0,\ j)=t,\quad \phi(0,\ j)=t \qquad \text{for all } i\\ &u(i\max,\ j)\to 0,\ v(0,\ j)\to 0,\ \theta(i\max,\ j)\to 0,\ \phi(i\max,\ j)\to 0 \qquad \text{for all } j \end{aligned} \tag{17}$$

Here the suffix i corresponds to y and j corresponds to t and $\Delta t = t(j+1) - t(j)$ and $\Delta z = Z(i+1) - Z(i)$. The computations are carried out for different values of the various physical parameters.

The procedure is repeated until the steady state. During computation Δt was chosen as 0.001. These computations are carried out for Pr= 0.71,1,7 and11 and for different values of Ec. To judge the accuracy of the convergence of the finite difference scheme, the same program was run with the Δt=0.0009, 0.00125 and no significant change was observed. Hence, we conclude the finite difference scheme is stable and convergent.

The physical quantities of primary interest are the local wall shear stress, the local surface heat and mass flux. Given the velocity field in the boundary layer, we can now calculate the local wall shear stress (i.e., skin – friction), is given in dimensionless form,

$$\tau = \frac{\tau_w}{\rho\ u_w^2} \text{ where } \tau_w = \left(\mu\ \frac{\partial u'}{\partial z'}\right)_{z'=0} = \rho\, v_0^2\ u'(0) \qquad \text{i.e. } \tau = -\left(\frac{\partial u}{\partial z}\right)_{Z=0} \tag{18}$$

Having known the temperature field, it is interesting to study the effect of the free convection and radiation on the rate of heat transfer. This is in dimensionless form as follows

$$Nu(x') = -\left[\frac{x'}{T'_w - T'_\infty}\left(\frac{\partial T'}{\partial z'}\right)_{z'=0}\right]$$

$$\text{i.e.}\qquad Nu = \frac{Nu(x')}{\mathrm{Re}_x} = -\left(\frac{\partial \theta}{\partial z}\right)_{Z=0} \tag{19}$$

From the definitions of the local mass flux and the local Sherwood number, respectively given, with the help of these equations, we can have the Sherwood number as

$$Sh(x') = -\left[\frac{x'}{C'_w - C'_\infty}\left(\frac{\partial C'}{\partial y'}\right)_{y'=0}\right]$$

$$\text{i.e. } Sh = \frac{Sh(x')}{\mathrm{Re}_x} = -\left(\frac{\partial \phi}{\partial z}\right)_{Z=0}$$

4. Results and Discussion

The effects of various parameters on the velocity, temperature and mass transfer are computed using numerical procedure.

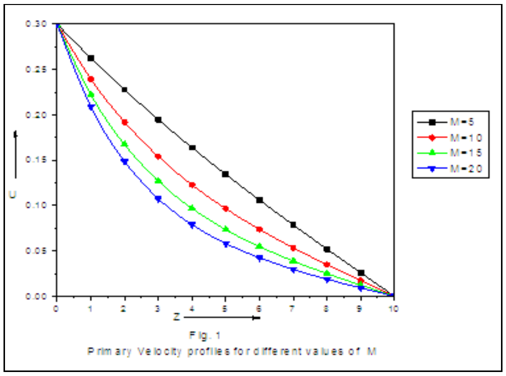

Figure 1

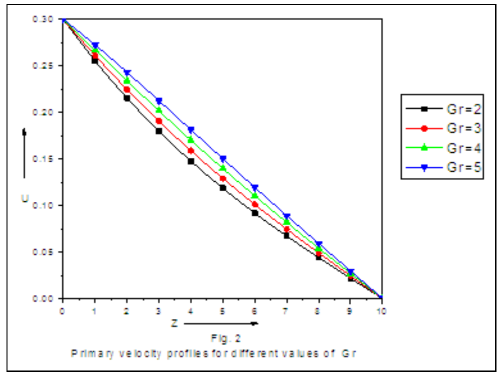

Figure 2

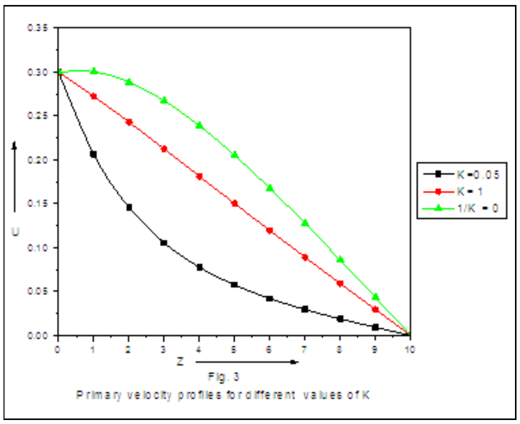

Figure 3

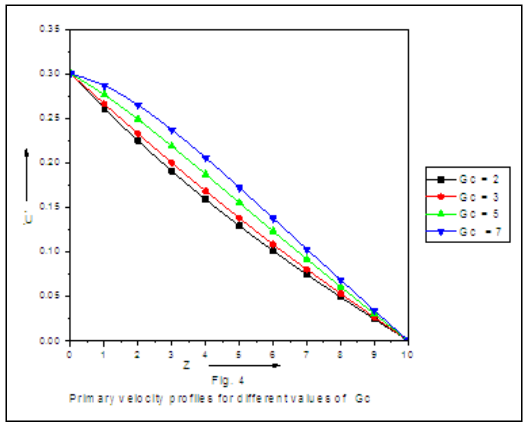

Figure 4

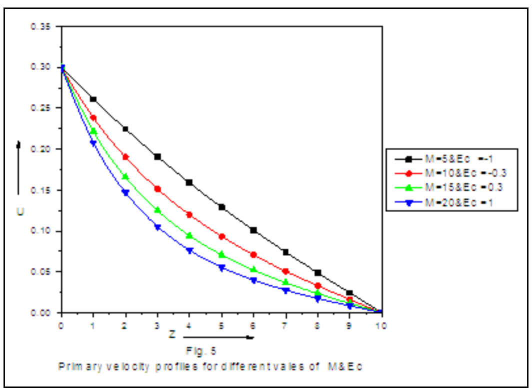

Figure 5

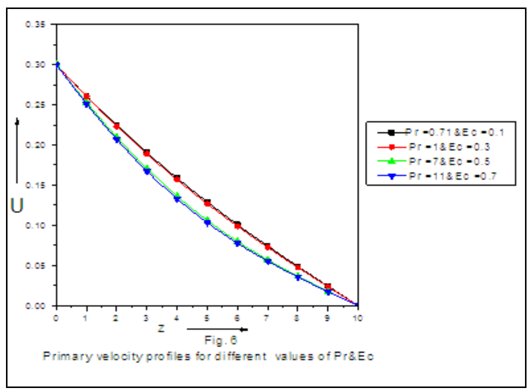

Figure 6

The primary velocity profiles are discussed through the graphs 1 to 6. The effect of the Hartmann number (M) is shown in the figure 1. The presence of a magnetic field in an electrically conducting fluid introduces the Lorentz force, which acts against the flow, results in the decrease of the velocity with the increasing of Hartmann number (M). From the figure, the decrease of velocity is observed with the increase of the magnetic field.

The variation of velocity distribution to the thermal Grashoff number is discussed in the figure 2. The thermal Grashoff number is the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. The positive values of Grashoff number indicate the cooling of the plate. The rise in the velocity is observed due to the enhancement of the thermal buoyancy force. As thermal buoyancy increases, simultaneously the velocity increases rapidly near the plate and gradually decreases to free stream velocity.

The increase of velocity distribution in the boundary layer is shown in the figure 3. This effect can be reduced when the holes of the porous medium is decreased.

The effect of Solutal Grashoff number is given in the figure 4. Similar to the thermal Grashoff number, the Solutal Grashoff number effect is also to increase in the velocity. The rise in the velocity distribution is observed in the figure.

The figures 5 and 6 show a comparative study for the Eckert number (Ec). As Eckert number increases, the velocity decreases in the presence of increasing magnetic field,

The secondary velocity profiles are presented for different parameters through the figures 7 to 12.

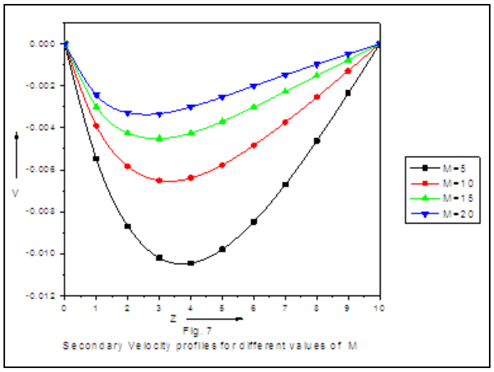

Figure 7

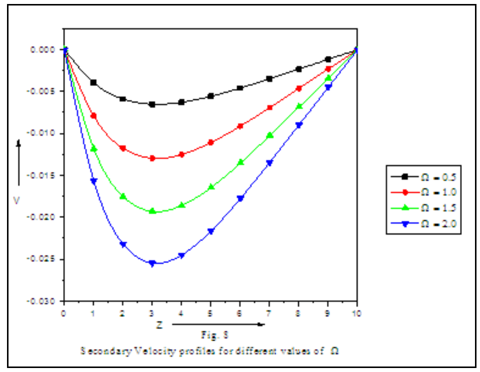

Figure 8

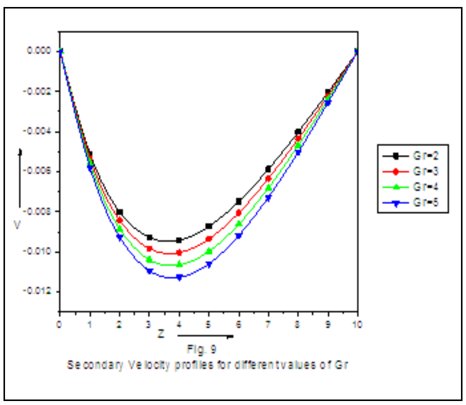

Figure 9

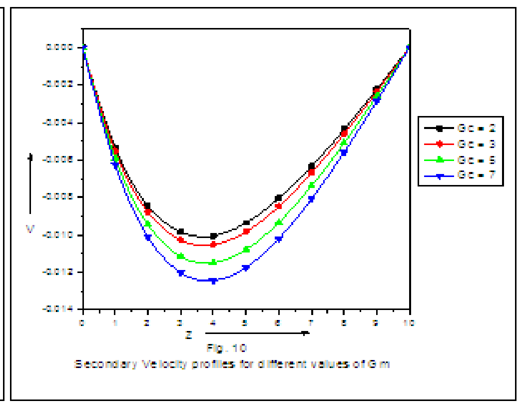

Figure 10

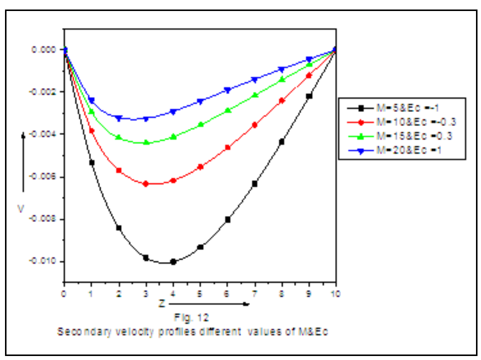

Figure 12

The effect of magnetic field (M) on the secondary velocity just reverses the process. This effect can be clearly observed from the figure 7. The secondary velocity profiles for different rotation parameters $\Omega = 0.5$ to 2.0 are shown in figure 8. The increase of velocity with decreasing values of the rotation parameter is clearly noticed from the figure.

The variation of velocity distribution to the thermal Grashoff number and Solutal Grashoff number is discussed in the figures 9 and 10. The thermal Grashoff number is the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. The positive values of Grashoff number indicate the cooling of the plate. The rise in the velocity is keenly observed due to the enhancement of the thermal buoyancy force. As thermal buoyancy increases, the velocity increases rapidly near the plate and gradually decreases to free stream velocity. Similar to the thermal Grashoff number, the Solutal Grashoff number effect is also leads to increase in the velocity. The rise in the velocity distribution is observed in the figure 10.

The increase of velocity distribution in the boundary layer is given in the figure 11. This effect can be reduced when the holes of the porous medium is decreased.

The comparative study of the effects of magnetic field (M) and Eckert number (Ec) is shown in the figure 12. The decrease in the velocity as the magnetic field increases can be observed from the comparative study.

The substantial change in the temperature of the flow and concentration distribution with parameters like Prandtl number and Eckert number are discussed through figures 13 to 15.

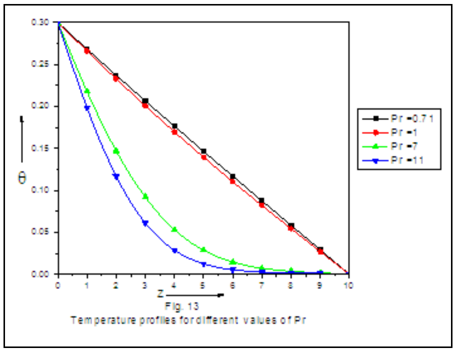

Figure 13

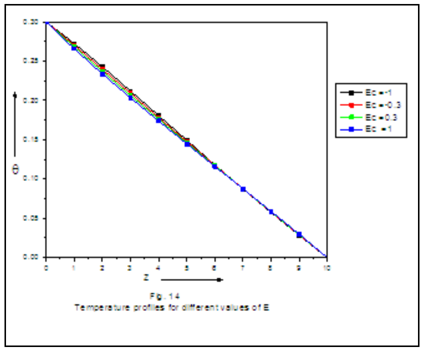

Figure 14

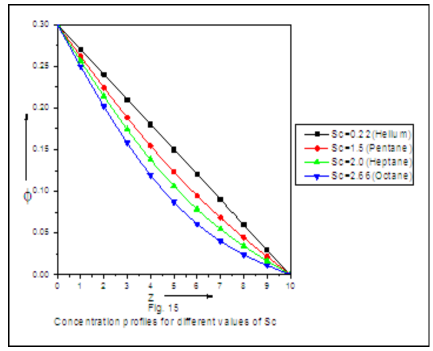

Figure 15

The decrease of temperature in the boundary layer is observed from the figure 13 with an increase in the Prandtl number. From the figure 14, a slight decrease in temperature is observed for increase in the values of Eckert number.

The concentration distribution is vastly affected by the presence of foreign species such as Helium (Sc = 0.22), Pentane (Sc = 1.49), Heptane (Sc = 2.0), and Octane (Sc = 2.66) which are given in the figure 15. The concentration is decreased with the increase of the Schmidt number i.e. with the presence of heavy foreign species.

Ω	Pr	M	Gr	Gc	K	Sc	Ec	τ (Skin friction)
0.5	0.71	5	3	2	1	0.22	0.3	-0.2248626
1	0.71	5	3	2	1	0.22	0.3	-0.2259557
0.5	7	5	3	2	1	0.22	0.3	-0.2475685
0.5	0.71	10	3	2	1	0.22	0.3	-0.233981
0.5	0.71	5	5	2	1	0.22	0.3	-0.2162077
0.5	0.71	5	3	4	1	0.22	0.3	-0.2120911
0.5	0.71	5	3	2	3	0.22	0.3	-0.2225116
0.5	0.71	5	3	2	1	0.66	0.3	-0.2285304
0.5	0.71	5	3	2	1	0.22	0.7	-0.2247407

Table 1: Skin Friction Coefficient

Table I shows numerical values of the Skin – friction coefficient for various values of rotation parameter (Ω), Prandtl Number (Pr), and Hartmann Number (M) etc.

From the table, it is primarily observed that when the rotation parameter, Pr, K and Sc increase, the shear stress decreases. With an increase in the Hartmann number, Eckert number, thermal Grashoff number, Solutal Grashoff number, the shear stress increases.

Ω	Pr	M	Gr	Gc	K	Sc	Ec	Nu (Nusselt number)
0.5	0.71	5	3	2	1	0.22	0.3	-3.516241 10^{-3}
1	0.71	5	3	2	1	0.22	0.3	-3.510094 10^{-3}
0.5	7	5	3	2	1	0.22	0.3	-7.096606 10^{-3}
0.5	0.71	10	3	2	1	0.22	0.3	-3.737477 10^{-3}
0.5	0.71	5	5	2	1	0.22	0.3	-3.633747 10^{-3}
0.5	0.71	5	3	4	1	0.22	0.3	-3.680822 10^{-3}
0.5	0.71	5	3	2	3	0.22	0.3	-3.548844 10^{-3}
0.5	0.71	5	3	2	1	0.66	0.3	-3.47737 10^{-3}
0.5	0.71	5	3	2	1	0.22	0.7	-3.673265 10^{-3}

Table 2: Variation of Nusselt Number

Table II gives the numerical values of the rate of heat transfer in terms of Nusselt number for various
Values of the rotation parameter, Hartmann number, and Eckert number etc. It is clear that with the increase in the magnetic field (M),Gr, K, Ω and Sc, results in increase of the rate of heat transfer. An increase in the Prandtl number (Pr) , Gc and Ec, decreases the rate of heat transfer.

Ω	Pr	M	Gr	Gc	K	Sc	Ec	Sh (Sherwood number)
0.5	0.71	5	3	2	1	0.22	0.3	-0.1999816
1	0.71	5	3	2	1	0.22	0.3	-0.1999791
0.5	7	5	3	2	1	0.22	0.3	-0.2151715
0.5	0.71	10	3	2	1	0.22	0.3	-0.2000114
0.5	0.71	5	5	2	1	0.22	0.3	-0.2000222
0.5	0.71	5	3	4	1	0.22	0.3	-0.2000329
0.5	0.71	5	3	2	3	0.22	0.3	-0.1999933
0.5	0.71	5	3	2	1	0.66	0.3	-0.2119681
0.5	0.71	5	3	2	1	0.22	0.7	-0.2000888

Table 3: Variation of rate of Mass Transfer

It is noticed that from table III, an increase in the rotation parameter, M, K and Ec causes to increase the value of mass transfer coefficient. For an increase in Pr, Gr, Gc and Sc, the mass transfer rate decreases.

5. Conclusion

The finite difference solution of an unsteady MHD free convective fluid flow of a viscous fluid past an accelerated vertical plate in the presence of rotation is thoroughly investigated. The plate temperature as well as concentration level near the plate are raised linearly with time. The effects of the various parameters on the velocity, temperature and concentration profiles are presented graphically and values of skin– friction coefficient, Nusselt number and Sherwood number for various values of physical parameters are also precisely discussed and finally conclude that:

- For an increase in M, M & Ec, Pr & Ec, the decrease will be in Primary velocity and for an increase in Gr, K, Gc the Primary velocity increases.
- With an increase in M, M & Ec, the secondary velocity increases, similarly the increase in Ω, Gr, Gc and K leads to the decrease in Secondary velocity.
- If Pr and Ec increase, the temperature will decrease.
- The increase in Sc results in the decrease of concentration.
- Skin friction coefficient increases with an increase in M, Gr, Gc and Ec and Skin friction coefficient decreases with the increase inΩ, Pr, K and Sc.
- Nusselt number increases respectively with the increase in Ω, M, Gr, K and Sc, and Nusselt number decreases with the increase in Pr, Gc and Ec.
- Mass transfer rate increases with an increase in Ω, M, K and Ec, as well as it decreases with the increase in Pr, Gr, Gc and Sc.

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