THE INTERNATIONAL JOURNAL OF SCIENCE & TECHNOLEDGE

A Study on Random Evolution Associated with A M/M/1∞ Queueing System with Balking

M. Reni Sagayaraj
Department of Mathematics, Sacred Heart College, Tirupattur, Vellore District, Tamil Nadu, India
S. Anand Gnana Selvam
Department of Mathematics, Sacred Heart College, Tirupattur, Vellore District, Tamil Nadu, India
R. Reynald Susainathan
Reckitt Benckiser, India Limited (Gurgaon), New Delhi. India

Abstract:
The arrival process in a queueing can be modified in several ways. One way is to allow arriving customers to leave the system without joining the queue. The act of a customer refusing to join the queue upon arrival is called balking. We consider an M/M/1/∞ queue with balking. The balking customers induce loss to the system and they alter the dynamics of the net profit. We associate various costs and analyze the Markov evolution of the net profit function. In this paper, we obtain the net profit function using the random evolution techniques.

***Key words**: Markov Evolution, Balking*

1. Introduction

We consider a counter where customers arrive according to a Poisson process with rate λ.There is a single server at the counter and he serves the customers according to the order of their arrival. The service time for a customer has exponential distribution with mean $\frac{1}{\mu}$. An arriving customer joins the queue with probability 1 if the system size is 0. If the system size is 1 when a customer's arrives, he joins the queue with probability p and he balks with probability q, $p+q=1$.

1.1. AN M/M/1/∞ Queue with Balking

For simplicity, we assume that at time t=0 there is 1 customer in the system. let $P_n(t)$ be the probability that there are n customers in the system at time t. Then $P_n(0)=\delta_{1,n.}$. We can easily obtain the forward equations as given below.

$$p_0'(t)=-\lambda p_0(t)+\mu p_1(t) \qquad (1)$$

$$p_1'(t)=-(\lambda p+\mu)p_1(t)+\lambda p_0(t)+\mu p_2(t) \qquad (2)$$

$$p_n'(t)=-(\lambda p+\mu)p_n(t)+\lambda p p_{n-1}(t)+\mu p_{n+1}(t),\ \ n\geq 2 \qquad (3)$$

We define

$$q_n(t)=\begin{cases} e^{(\lambda p+\mu)t}[\mu p_n(t)-\lambda p p_{n-1}(t)],\ n\geq 2 \\ e^{(\lambda+\mu)t}[\mu p_1(t)-\lambda p_0(t)],\ \ n-1 \\ 0 \qquad\qquad n=0,-1,-2,.... \end{cases} \qquad (4)$$

then , for n ≥ 3, we have

$$\begin{aligned} q_n'(t) &= (\lambda p+\mu)q_n(t)+e^{(\lambda p+\mu)t}[\mu\{-(\lambda p+\mu)p_n(t)+\lambda p p_{n-1}(t)+\mu p_{n+1}(t)\} \\ &\quad -\lambda p\{-(\lambda p+\mu)p_{n-1}(t)+\lambda p p_{n-2}(t)+\mu p_n(t)\}] \\ &= \mu q_{n+1}(t)+\lambda p q_{n-1}(t) \qquad (5) \end{aligned}$$

For n = 2, we have

$$q'_2(t) = (\lambda p + \mu) q_2(t) + e^{(\lambda p+\mu)t}[\mu\{-(\lambda p + \mu) p_2(t) + \lambda p p_1(t) + \mu p_3(t)\}$$
$$- \lambda p\{-(\lambda p + \mu) p_1(t) + \lambda p_0(t) + \mu p_2(t)\}]$$
$$= \mu q_3(t) + \lambda p q_1(t) \qquad (6)$$

$$q'_1(t) = (\lambda + \mu) q_2(t) + e^{(\lambda+\mu)t}[\mu\{-(\lambda p + \mu) p_1(t) + \lambda p_0(t) + \mu p_2(t)\}$$
$$- \lambda\{\lambda p_0(t) + \mu p_1(t)\}]$$
$$= \mu q_2(t) \qquad (7)$$

The Equations (5), (6), and (7) are subject to the initial condition

$$q_n(0) = \begin{cases} \mu \ , n-1 \\ -\lambda p \ , \ n = 2 \\ 0 \ , otherwise \end{cases} \qquad (8)$$

Defining

$$H(s,t) = \sum_{n=-\infty}^{\infty} q_n(t) s^n$$

We get by using the equations (5),(6) and (7)

$$\frac{\partial H(s,t)}{\partial t} = \left(\frac{\mu}{s} + \lambda ps\right) H(s,t) - \mu q_1(t) \qquad (9)$$

Subject to the condition $H(s,0) = s[\mu - \lambda ps]$. keeping $q_1(t)$ as un known (9) can be resolved and we obtain

$$H(s,t) = H(s,0)\exp\left\{\left(\frac{\mu}{s} + \lambda ps\right)t\right\} - \mu\int_0^t \exp\left\{\left(\frac{\mu}{s} + \lambda ps\right)(t-u)\right\} q_1(u)du \qquad (10)$$

Setting $\lambda p = \frac{\alpha\beta}{2}$ *and* $\mu = \frac{\alpha}{2\beta}$, *we have* $\alpha = \sqrt{\lambda p \mu}$ *and* $\beta = \sqrt{\frac{\lambda p}{\mu}}$ *sothat*

$$\exp\left\{\left(\frac{\mu}{s} + \lambda ps\right)t\right\} = \exp\left\{\frac{1}{2}\left((\beta s) + \frac{1}{(\beta s)}\right)(\alpha t)\right\} = \sum_{-\infty}^{\infty} (\beta s)^n I_n(\alpha t) \qquad (11)$$

Where $I_n(\alpha t)$, n=0,±1,±2,....... are modified Bessel's functions of the first kind given by

$$I_n(u) = \sum_{k=0}^{\infty} \frac{u^{n+2k}}{2^{n+2k} k!(n+k)!}, n > -1; I_{-n}(u) = I_n(u)$$

Substituting (11) in (10), We get

$$\sum_{n=-\infty}^{\infty} q_n(t) s^n = s[\mu - \lambda ps] \sum_{-\infty}^{\infty} (\beta s)^n I_n(\alpha t) - \mu \int_0^t \sum_{-\infty}^{\infty} (\beta s)^n I_n(\alpha(t-\mu)) q_1(u)du \qquad (12)$$

Equating the coefficient of s^n in (12) for n = 1,2,.... we obtain

$$q_n(t) = \mu\beta^{n-1} I_{n-1}(\alpha t) - \lambda p \beta^{n-2} I_{n-2}(\alpha t) - \mu\beta^n \int_0^t I_n(\alpha(t-\mu)) q_1(u)\,du \qquad (13)$$

1.2. The Markov Evolution Of A Net Profit Function

Let r(t) be the value of the service per unit time per customer in the system at time t. We define

$$r(t) = \begin{cases} -r_1 & if\ n(t) = 0 \\ r_2 & if\ n(t) = 1 \\ r_3 & if\ n(t) > 0 \end{cases} \qquad (14)$$

Where n(t) represents the number of customers in the system at time t. Here we assume that $r_i > 0$; i = 1, 2, 3. It is to be noted that $-r_1$ is the negative cost due to the idle time of the server and r_2, r_3 correspond to the state dependent positive gain due to the customers who joined the system. Clearly r_2 and r_3 contribute positive revenue to the net profit function. Then the net profit function L(t) is given by the stochastic integral.

$$L(t) = \int_0^t r(u)\,du \qquad (15)$$

In [15], by taking r(u) as the instantaneous velocity of the server , L(t) gives the distance travelled by server in time t. Then the time evolution of the net gain can be studied by identifying the server as a particle under a random motion [12] on the real line with three velocities in a cyclic manner. We now study the time evolution of L(t). For this , we assume that the server enters into the idle state at time t=0 . Then we note that n (o = 0 and r(0) = $-r_1$ .It is easy to note that the discrete component of L(t) is given by

$$\Pr\{L(t) = -r_1(t)\} = e^{-\lambda t} \qquad (16)$$

2. Conclusion

We have proposed a varies cost analyze the Markov evolution of the net profit.The balking behaviour is incorporated by assuming that an arriving customer joins the queue with probability p and the balks with probability q.Explicit expression for the transient probabilities $P_n(t)$ are found in a direct way along with steady state solution. The model extends substantially the earlier works available in the literature.

3. References

1. [Abate.J. and W.Whitt (1987): Transient bahavior of the M/M/1 queue: Starting at the origin,Queueing Systems, Vol. 2,41-65.
2. Abate.J. and W.Whitt (1988): Transient bahavior of the M/M/1 queue via Laplace Transforms, Adv.Appl.Probab., Vol.20, 145-178.
3. Bartlett, M. (1978): A note on random walks at constant speed, Adv. Appl. prob. Vol. 10, pp. 704-707.
4. Cane, V. (1967): Random walks and Physical Processes, Bull. Int. Statst. Inst., Vol. 42, pp.622-640.
5. Cohen, J.W. (1982) : The single server Queue, North-Holland, Amsterdam.
6. Di Crescenzo,A. (2001) : On random Motions with Velocities alternating at Erlang distributed random times, Adv . Appl . Proob., Vol. 33, pp.690-701.
7. Dunbar, S. R. (1988): A branching random evolution and a nonlinear hyperbolic equation, SIAM J. Appl.Math., Vol. 48, pp. 1510-1526.
8. Foong, S.K. and and Kanno, S. (1994) : Properties of telegrapher's random process with or without a trap,Stoch. Proc. Appl., Vol. 53, pp. 147-173.
9. Goldstein, S. (1951): On di_usion by discontinuous movements and the telegraph equation, Quart. J.Mech.Appl.Math., Vol. 4, pp.129-156.
10. Kac, M. (1974): A Stochastic model related to the telegrapher's equation, Rocky Mountain J.Math. Vol. 4, pp.497-509.
11. Kleinrock, L. (1975): Queueing systems Vol.1: Theory, J. Wiley, New York.
12. Klesnik, a. (1998) : The equations of Markovian random evolution on the line, J. Appl. Prob. Vol.35, pp.27-35.
13. Korolyuk, V.S. and Korolyuk, V.V. (1999):Stochastic Models of Systems, Kluwer Academic publishers.
14. Krishnakumar, B. and Arivudainambi, D.(2000): Transient solution of an M/M/1 queue with catastrophes, computers and mathematics with Applications, Vol. 40, pp. 1233-1240.
15. Krishnakumar, B.,Parthasarathy,P.R. and Sharafali,M(1993)., Transient solution of an M/M/1 queue with balking , Queueing system(Theory and Applications) Vol.13, pp.441-447